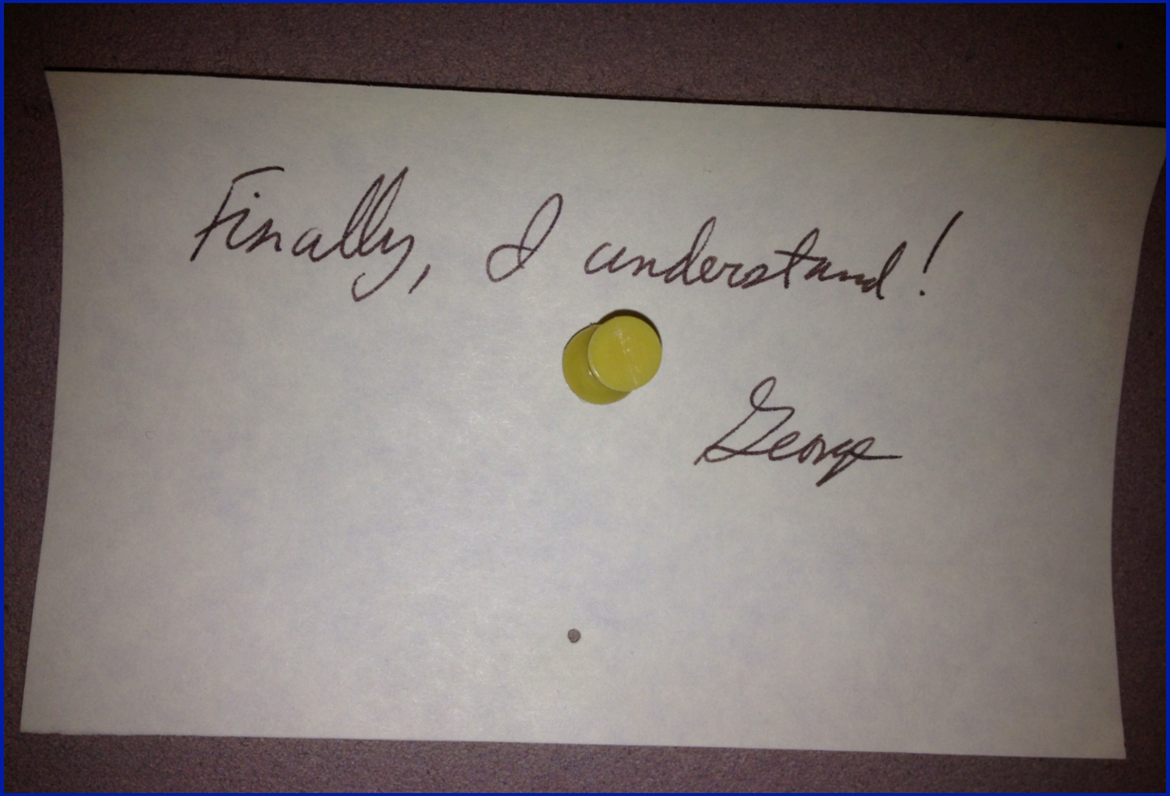
The Bertsch parameter, the unitary Fermi gas, and all that follow-up

(well, … lots of that anyway)

Aurel Bulgac
University of Washington



What are the ground state properties of the many-body system composed of spin ½ fermions interacting via a zero-range, infinite scattering length contact interaction.

Bertsch's Many-Body X challenge, Seattle, 1999

This is a slightly idealized model for dilute neutron matter with an attractive two-body interaction. In neutron star crust $k_F|a| = O(10)$ and $|a|/r_0 \approx O(10)$ and this is a strongly interactive Fermi system and naïve models fail.

$$r_0 \ll n^{-1/3} \approx \frac{\pi}{k_F} \ll |a|$$

$$\sigma(k) = \frac{4\pi a^2}{1+k^2a^2} \approx \frac{4}{\pi n^{2/3}}$$

Let us consider a very old and simple example:
the hydrogen atom.

The ground state energy could only be a function of:

- ✓ Electron charge
- ✓ Electron mass
- ✓ Planck's constant

and then trivial dimensional arguments lead to

$$E_{gs} = \frac{e^4 m}{\hbar^2} \times \frac{1}{2}$$

Only the factor ½ requires a serious theory.

Let us turn now to dilute fermion matter

The ground state energy is given by a function:

$$E_{gs} = f(N, V, \hbar, m, a, r_0)$$

Taking the scattering length to infinity and the range of the interaction to zero, we are left with:

Pure number

$$E_{gs} = F(N, V, \hbar, m) = \frac{3}{5}\varepsilon_F N \times \boxed{\xi}$$

if $\xi > 0$ - the system is a gas with positive presure

if $\xi < 0$ - the system collapses, since presure is negative

$$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m}, \quad n = \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

George actually wanted to know the sign of ξ.

In 1999 we did not know the sign of ξ!

There were a number of papers making opposite claims around that time.

- **G.A. Baker, Jr (LANL) won the $600 prize**

($300 from George + $300 from V.A. Khodel)

Phys. Rev. C 60, 064901 (1999)

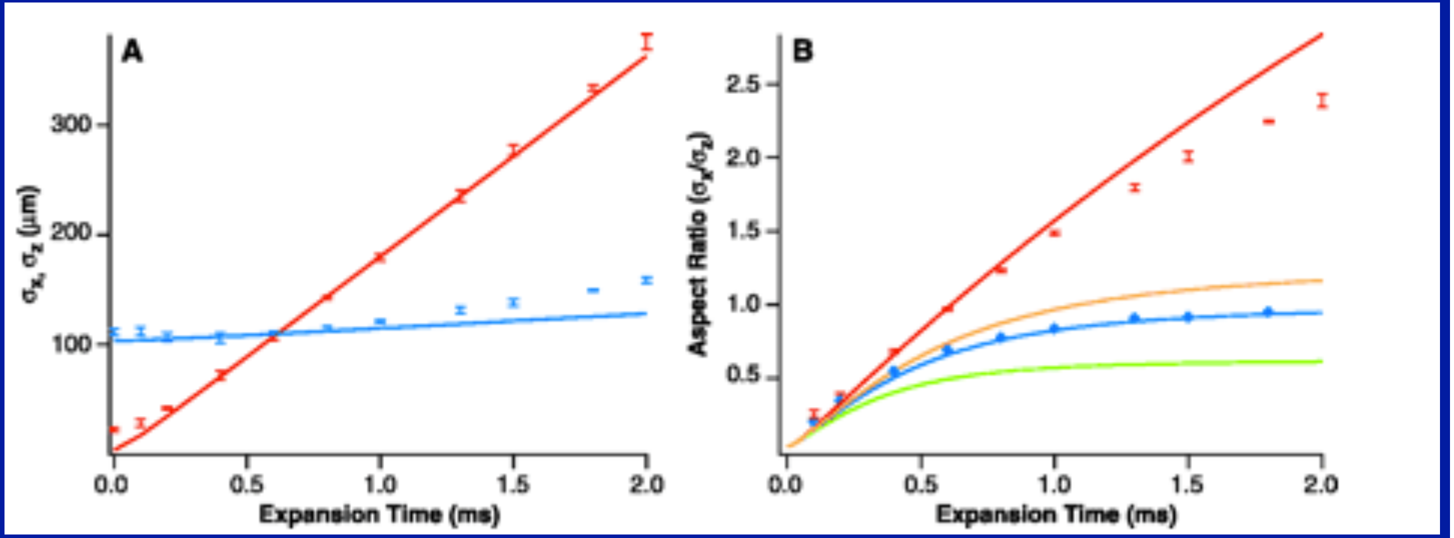
The Bertsch, nonparametric model of neutron matter is analyzed and strong indications are found that, in the infinite system limit, the ground state is a Fermi liquid with an effective mass, except for a set of measure zero.

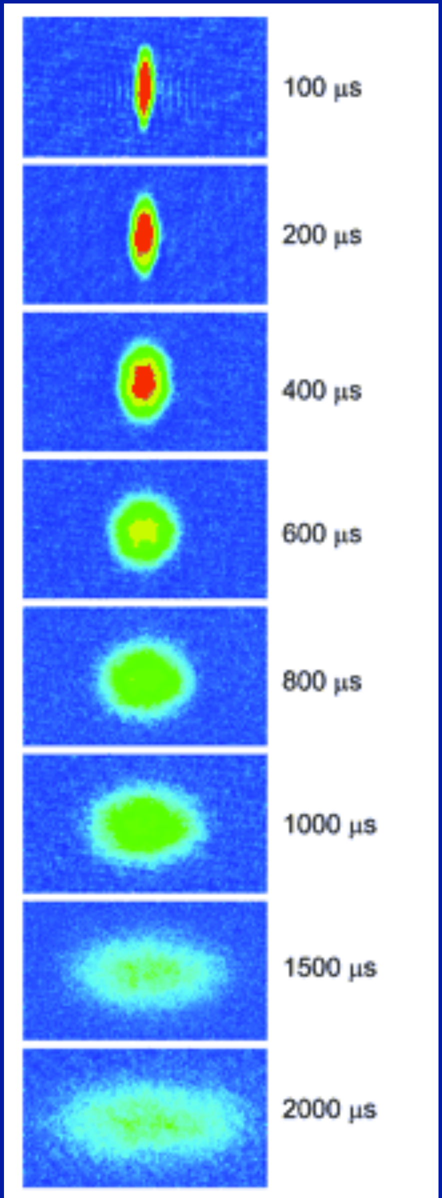
- **H. Heiselberg, second runner-up**

Phys. Rev. A 63, 043606 (2001)

Ground-state energies and superfluid gaps are calculated for degenerate Fermi systems interacting via long attractive scattering lengths such as cold atomic gases, neutron, and nuclear matter. In the Intermediate region of densities, where the interparticle spacing (~$1/k_F$) is longer than the range of the interaction but shorter than the scattering length, the superfluid gaps and the energy per particle are found to be proportional to the Fermi energy and thus differ from the dilute and high-density limits. The attractive potential increase linearly with the spin-isospin or hyperspin statistical factor such that, e.g., symmetric nuclear matter undergoes spinodal decomposition and collapses whereas neutron matter and Fermionic atomic gases with two hyperspin states are mechanically *stable* in the intermediate density region. The regions of spinodal instabilities in the resulting phase diagram are reduced and do not prevent a superfluid transition.

Observation of a Strongly Interacting Degenerate Fermi Gas of Atoms
O'Hara, Hemmer, Gehm, Granade, and Thomas
Science, 298, 2179 (2002)





The atomic cloud expansion is similar to that observed in RHIC heavy-ion collisions.

Superfluid Fermi Gases with Large Scattering Length

Carlson, Chang, Pandharipande, and Schmidt

Phys. Rev. Lett. 91, 050401 (2003)

We report quantum Monte Carlo calculations of superfluid Fermi gases with short-range two-body attractive interactions with infinite scattering length. The energy of such gases is estimated to be ***0:44 ±0:01*** *times that of the non-interacting gas, and their pairing gap is approximately twice the energy per particle.*

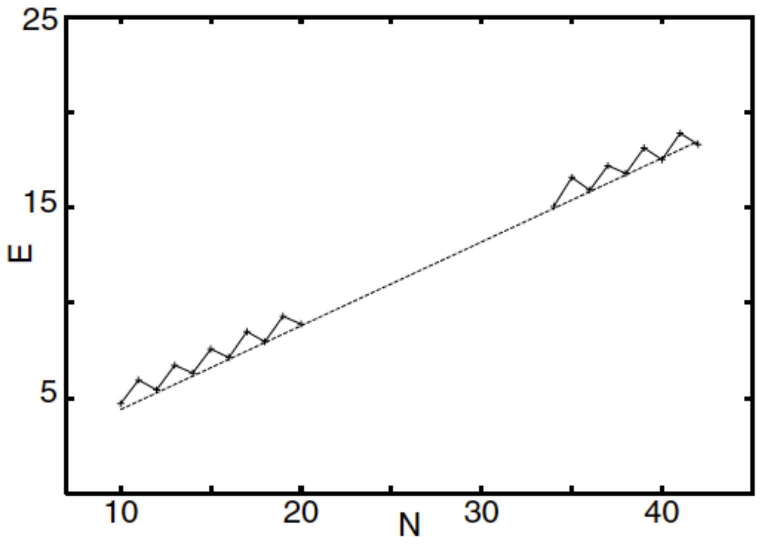


FIG. 3. The $E(N)$ in units of E_{FG}.

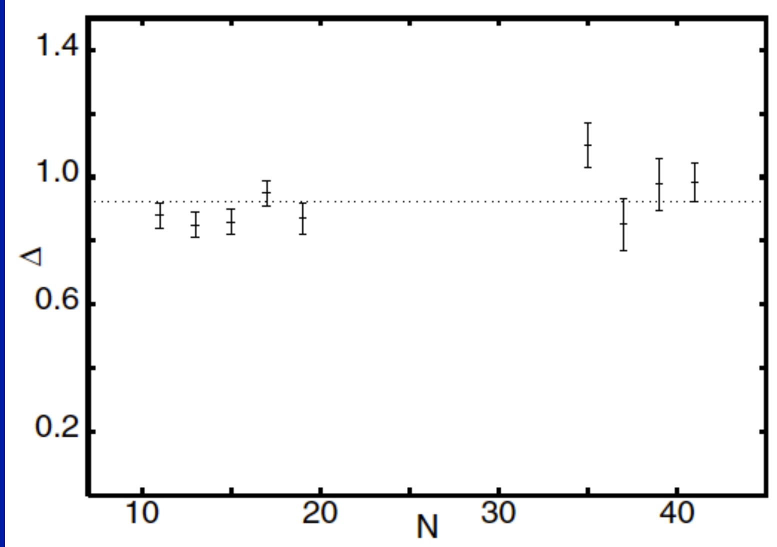
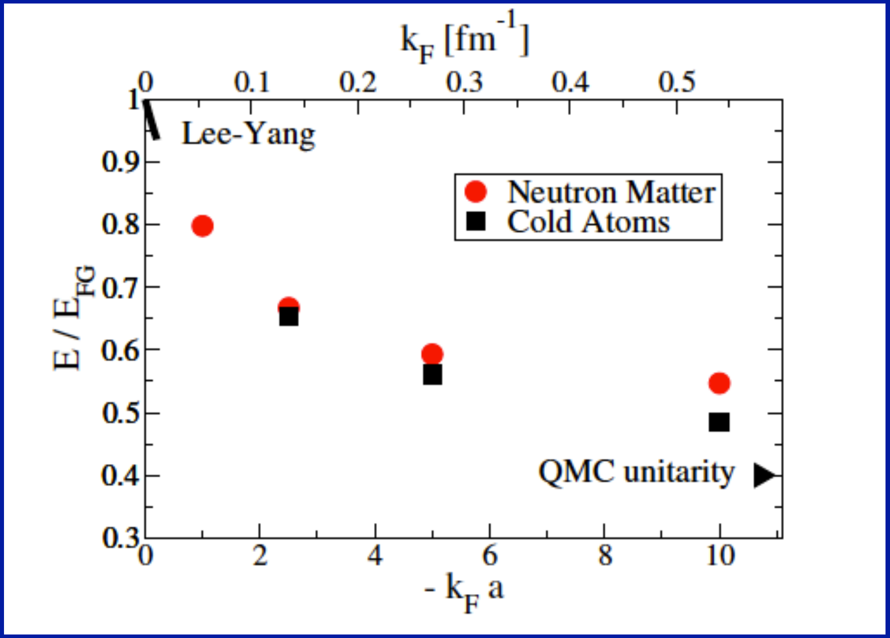


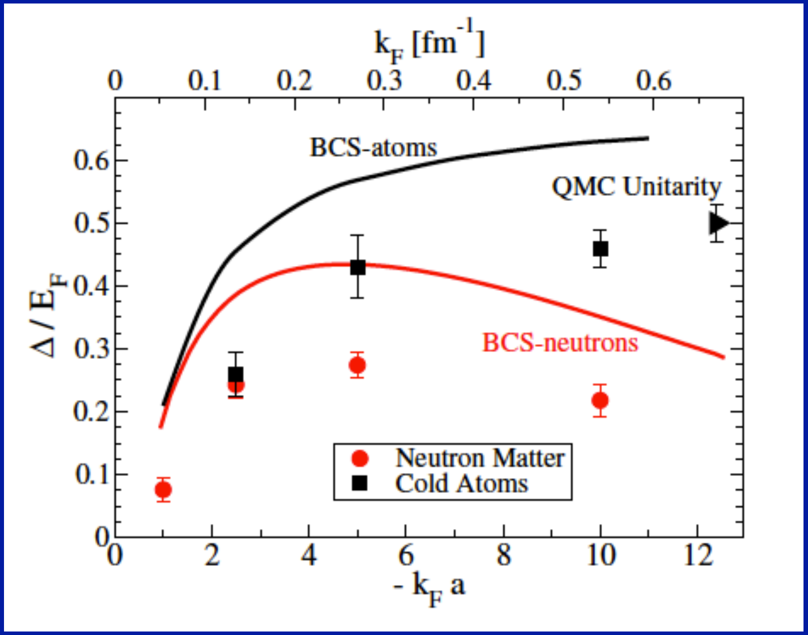
FIG. 4. The gap in units of E_{FG}.

$$E_{FG} = \frac{E}{N} = \frac{3\hbar^2 k_F^2}{10m}, \qquad n = \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

Superfluid pairing in neutrons and cold atoms

Carlson, Gandolfi, and Gezerlis, arXiv:1204.2596





$$E_{FG} = \frac{E}{N} = \frac{3\hbar^2 k_F^2}{10m}, \qquad n = \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

Institutions working with atomic Fermi gases

Institute, country	Head of laboratory	Atom	Year of the first result, reference
JILA, USA	Deborah Jin	^{40}K	1999 [1]
Rice Univ., USA	Randall Hulet	^{6}Li	2001 [71]
Ecole Normale Supérieure, France	Christophe Salomon	^{6}Li	2001 [72]
Duke Univ., then North Carolina State Univ., USA	John Thomas	^{6}Li	2002 [66]
MIT, USA	Wolfgang Ketterle	^{6}Li	2002 [73]
Univ. Firenze, Italy	Massimo Inguscio	^{40}K	2002 [74]
Univ. Innsbruck, Austria	Rudolf Grimm	^{6}Li, ^{40}K	2003 [75]
Eidgenössische Technische Hochschule, Switzerland	Tilman Esslinger	^{40}K, ^{6}Li	2005 [76]
Tübingen Univ., Germany	Claus Zimmermann	^{6}Li	2005 [77]
Vrije Univ., The Netherlands	Wim Vassen	^{3}He	2006 [78]
Kyoto Univ., Japan	Yoshiro Takahashi	^{173}Yb, ^{171}Yb, ^{6}Li	2007 [79]
Swinburne Univ. Technology, Australia	Christopher Vale	^{6}Li	2007 [80]
Univ. Electro-Communications, Japan	Takashi Mukaiyama	^{6}Li	2008 [81]
Max Planck Inst. Kernphysik, then Univ. Heidelberg, Germany	Selim Jochim	^{6}Li	2008 [15]
Pennsylvania State Univ., USA	Kenneth O'Hara	^{6}Li	2009 [82]
MIT, USA	Martin Zwierlein	^{6}Li, ^{40}K	2009 [83]
Institute of Applied Physics, Russian Academy of Sciences, Russia	Andrey Turlapov	^{6}Li	2010 [16]
Rice Univ., USA	Thomas Killian	^{87}Sr	2010 [84]
Univ. Cambridge, United Kingdom	Michael Köhl	^{40}K	2011 [85]
Univ. Washington, USA	Subhadeep Gupta	^{6}Li	2011 [86]

Turlapov, JETP Lett. 95, 96 (2012)

Vortices and Superfludity in a strongly interacting Fermi gas

Zwierlein, Abo-Shaeer, Schirotzek, Schunck, and Ketterle, Nature <u>435</u>, 1047(2005)

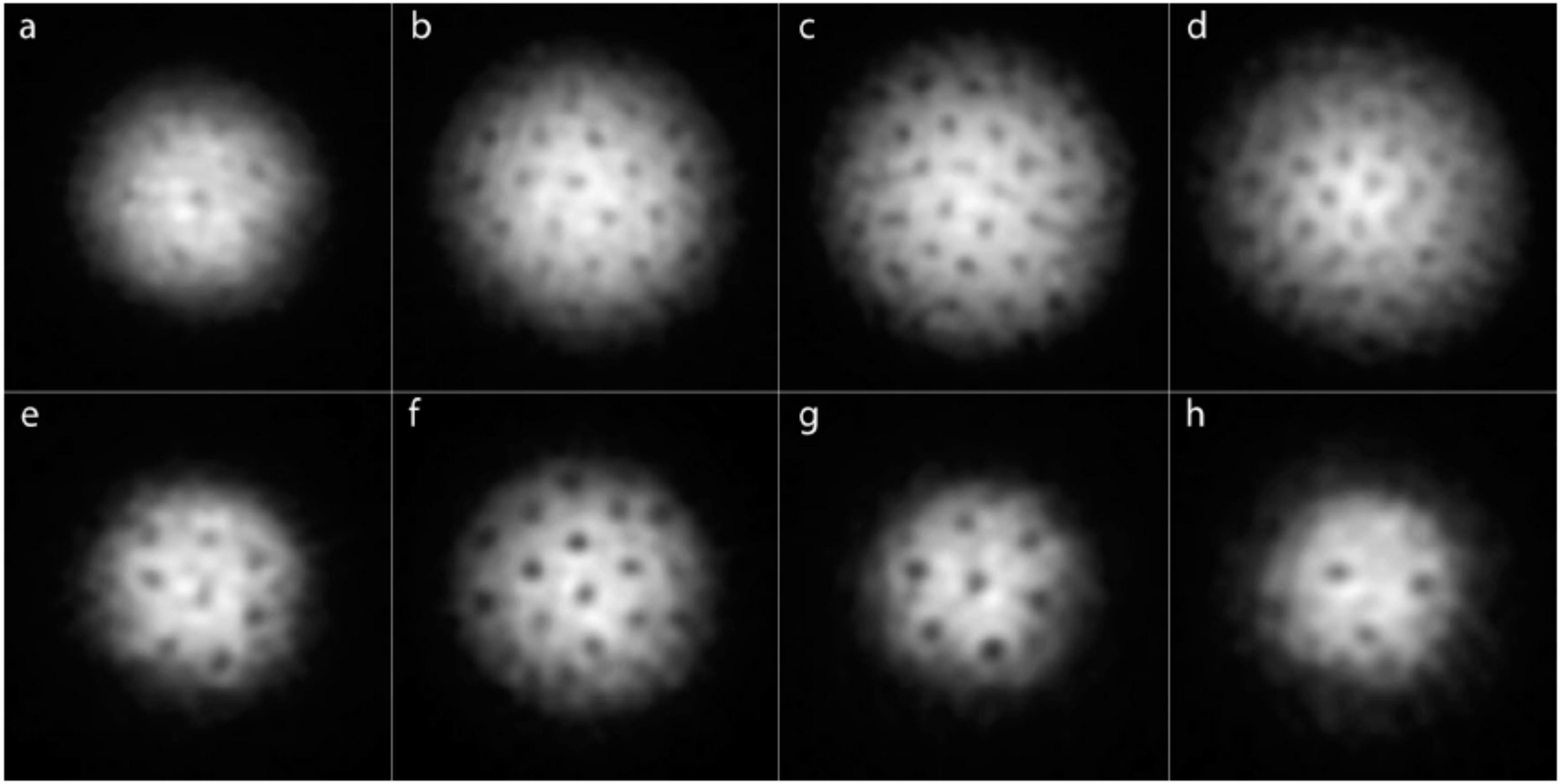
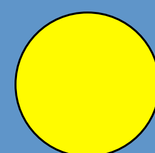
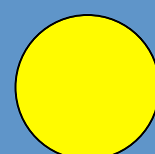
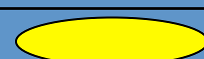


Fig. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G and (h) 863 G. The field of view of each image is $880\ \mu m \times 880\ \mu m$.

Sound in infinite fermionic matter

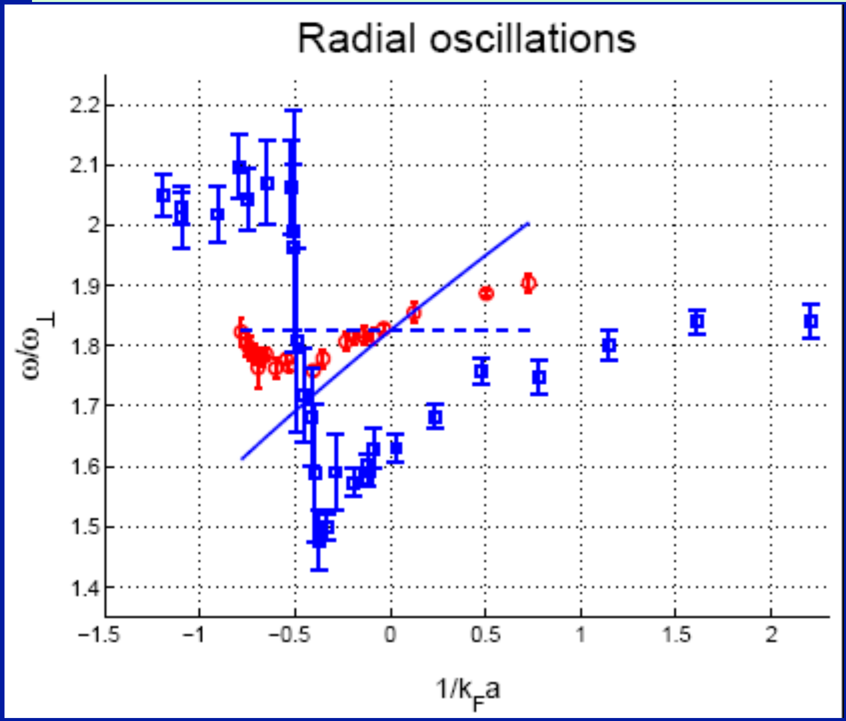
$$\omega = \mathrm{v}_s k$$

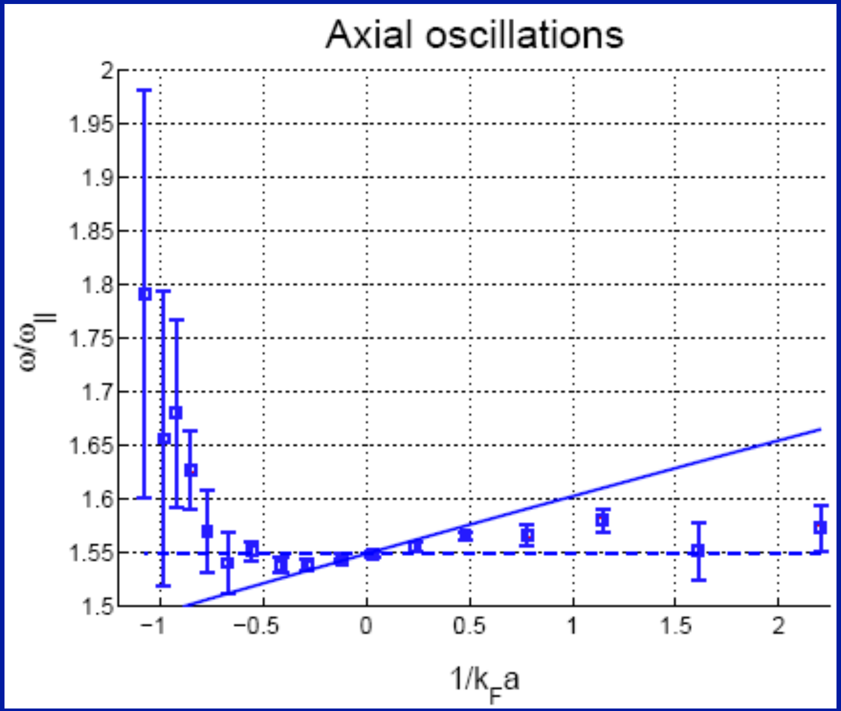
	Local shape of Fermi surface	Sound velocity	
Collisional Regime - <u>high T!</u> Compressional mode	Spherical 	$\mathrm{v}_s \approx \frac{\mathrm{v}_F}{\sqrt{3}}$	First sound
Superfluid collisionless- <u>low T!</u> Compressional mode	Spherical 	$\mathrm{v}_s \approx \frac{\mathrm{v}_F}{\sqrt{3}}$	Bogoliubov-Anderson sound
Normal Fermi fluid collisionless - <u>low T!</u> (In)compressional mode	 Elongated along propagation direction	$\mathrm{v}_\mathrm{s} = s\mathrm{v}_\mathrm{F}$ $s > 1$	Landau's zero sound Need repulsion !!!

Away form unitarity

$$\varepsilon(n)=\frac{E}{N}=\frac{3\hbar^2k_F^2}{10m}\left[\xi-\frac{\varsigma}{k_Fa}-\frac{5\iota}{(k_Fa)^2}+O\left(\frac{1}{(k_Fa)^3}\right)\right],\qquad \xi\approx 0.44,\quad \varsigma\approx 1,\quad \iota\approx 1$$

$$U=\frac{m\omega_0^2\left(x^2+y^2+\lambda^2z^2\right)}{2},\quad \frac{\delta\omega^2}{\omega^2}=\frac{\varsigma}{\xi}\frac{1}{k_F(0)a}K,\qquad K-\text{ parameter determined by cloud shape}$$





First order perturbation theory prediction (blue solid line)

Bulgac and Bertsch, Phys. Rev. Lett. 94, 070401 (2005)

Non-interacting fermions (blue dashed line)

Initial Innsbruck's data (later corrected) - blue symbols, Duke's data - red symbols

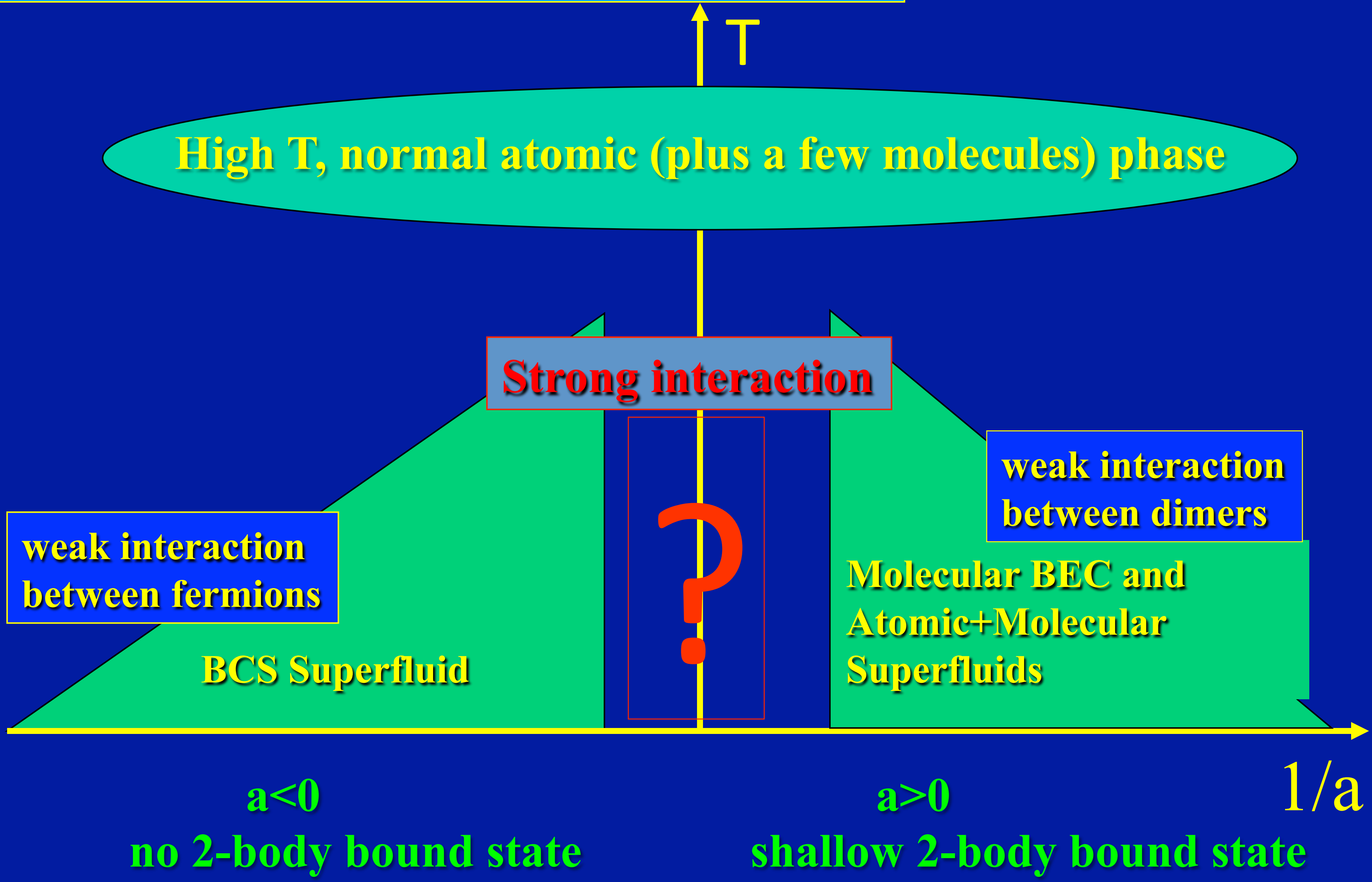
Why should one study fermionic superfluidity?

Superconductivity (which turned 100 years old on April 8th, 2011) and superfluidity in Fermi systems are manifestations of quantum coherence at a macroscopic level

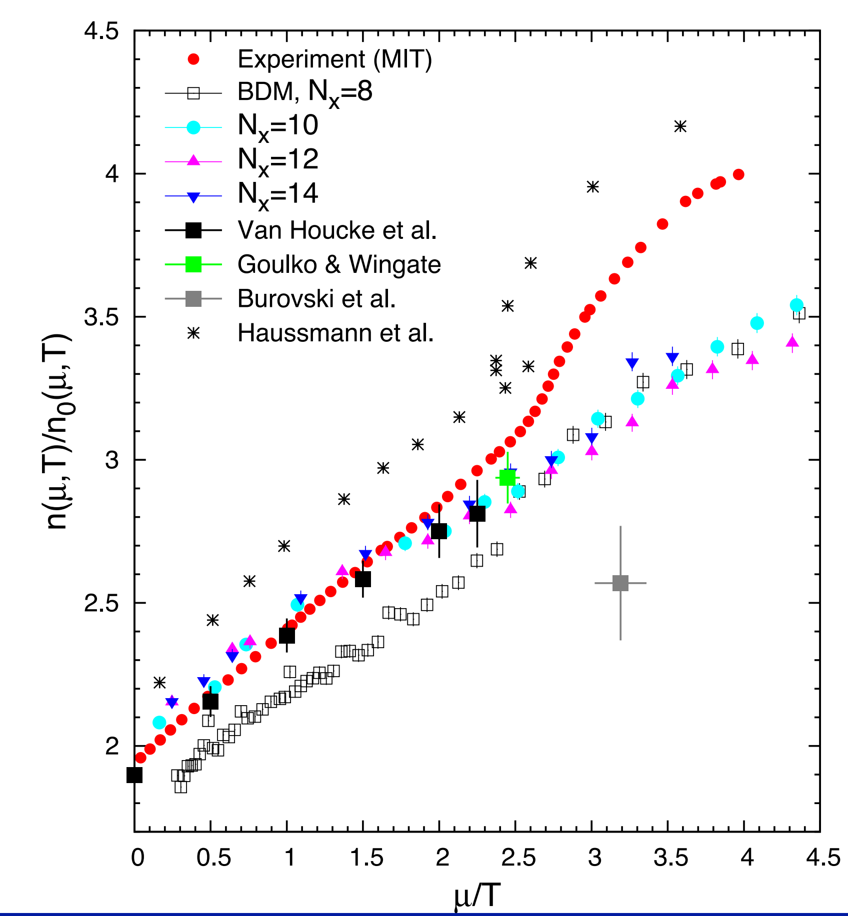
✓ Dilute atomic Fermi gases	$T_c \approx 10^{-9}$ eV
✓ Liquid ^{3}He	$T_c \approx 10^{-7}$ eV
✓ Metals, composite materials	$T_c \approx 10^{-3} - 10^{-2}$ eV
✓ Nuclei, neutron stars	$T_c \approx 10^{5} - 10^{6}$ eV
• QCD color superconductivity	$T_c \approx 10^{7} - 10^{8}$ eV

units (1 eV ≈ 10^4 K)

Phases of a two species dilute Fermi system in the BCS-BEC crossover



Theory versus experiment for Equation of State



Ku, Sommer, Cheuk, and Zwierlein, Science, 335, 563 (2012)
Bulgac, Drut, and Magierski – (BDM, N_x= 8), Phys. Rev. Lett. 96, 090404 (2006)
Burovski, Prokofiev, Svistunov, and Troyer, Phys. Rev. Lett. 96, 160502 (2006)
Drut, Lahde, Wlazlowski, and Magierski – (N_x= 10, 12, 14), Phys. Rev. A 80, 051601(R) (2012)
Goulko and Wingate, Phys. Rev. A 82, 053621 (2010)
Van Houcke, … , Zwierlein, Nature Physics, 8, 366 (2012)
Haussmann and Zwerger, Phys. Rev. A 78, 063602 (2008)

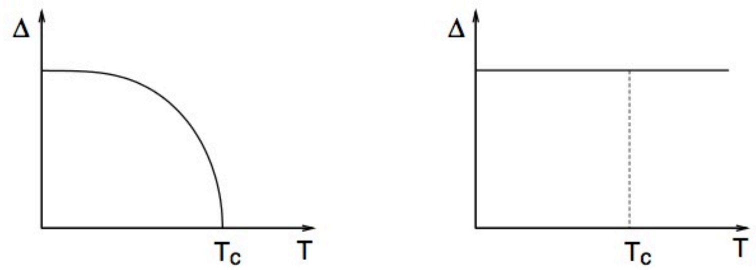
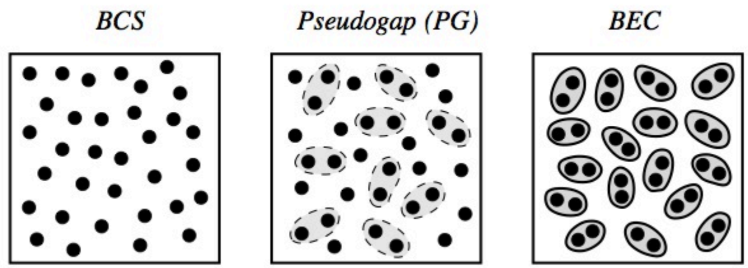


FIG. 4 Comparison of typical temperature dependences of the excitation gaps in the BCS (left) and BEC (right) limits. For the former, the gap is small and vanishes at T_c; whereas for the latter, the gap is very large and essentially temperature independent.



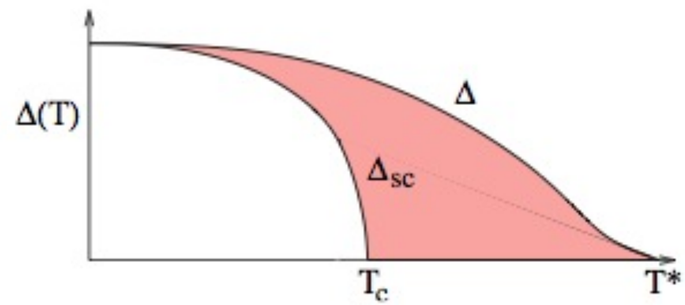
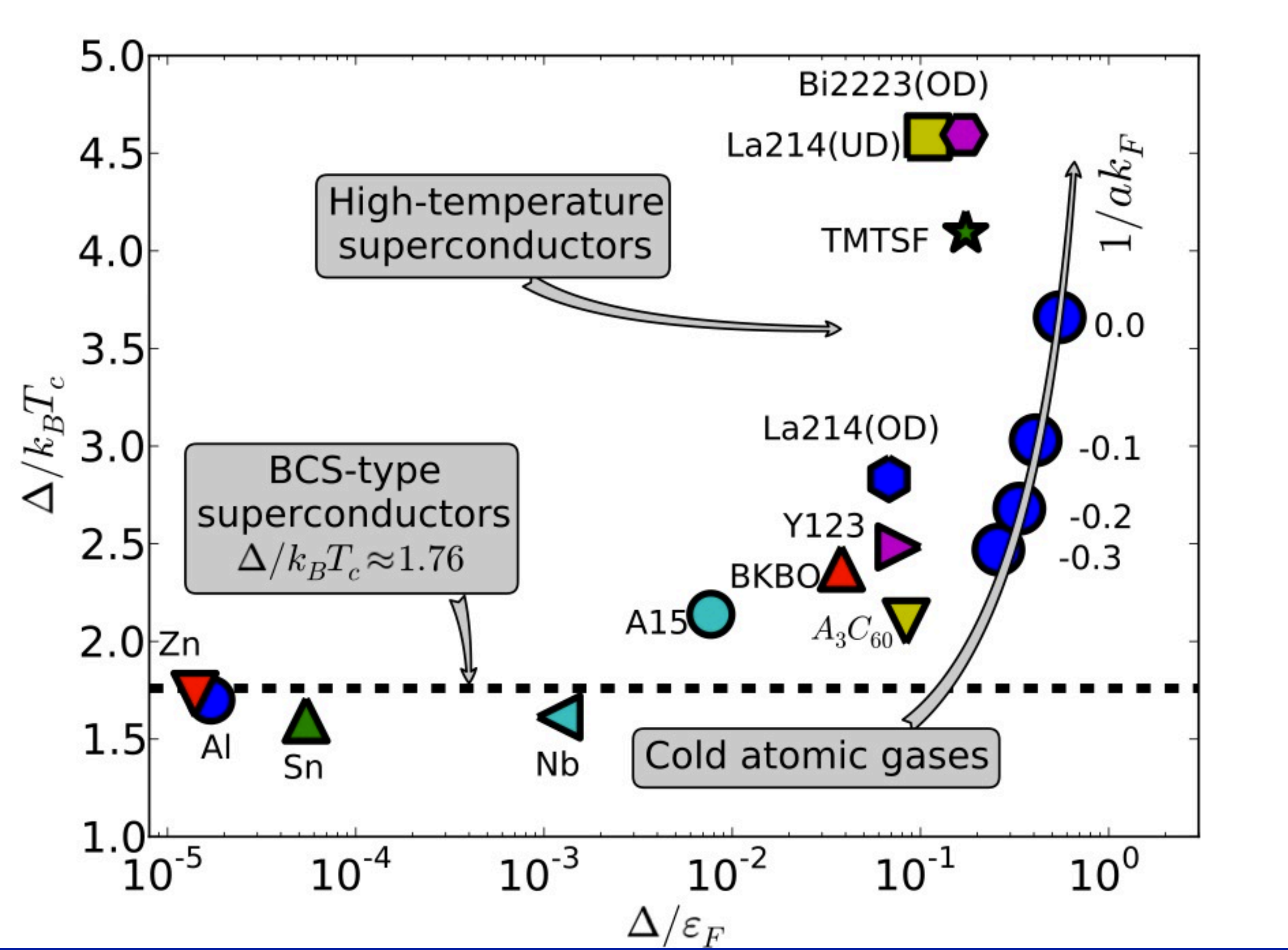


FIG. 3 Contrasting behavior of the excitation gap $\Delta(T)$ and superfluid order parameter $\Delta_{sc}(T)$ versus temperature. The height of the shaded region roughly reflects the density of noncondensed pairs at each temperature.

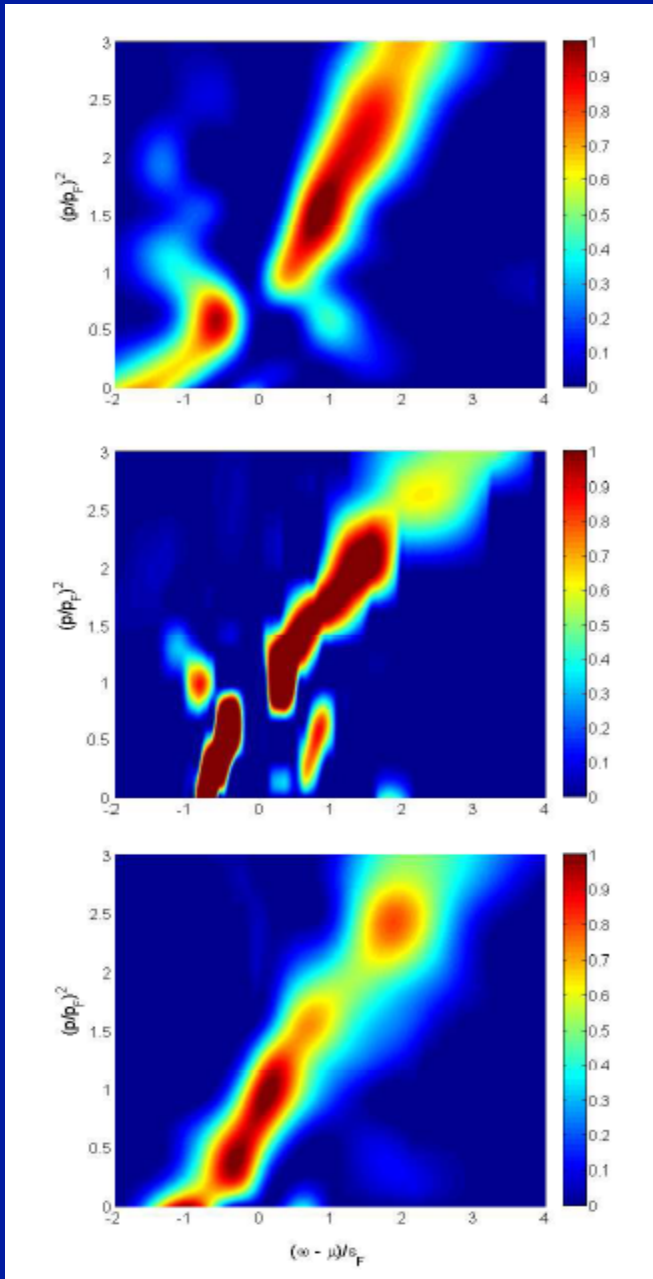
$T > T_c$

Chen, Stajic, Tan, and Levin, Phys. Rep. 412, 1 (2005)

Unitary Fermi gases are unconventional fermionic superfluids

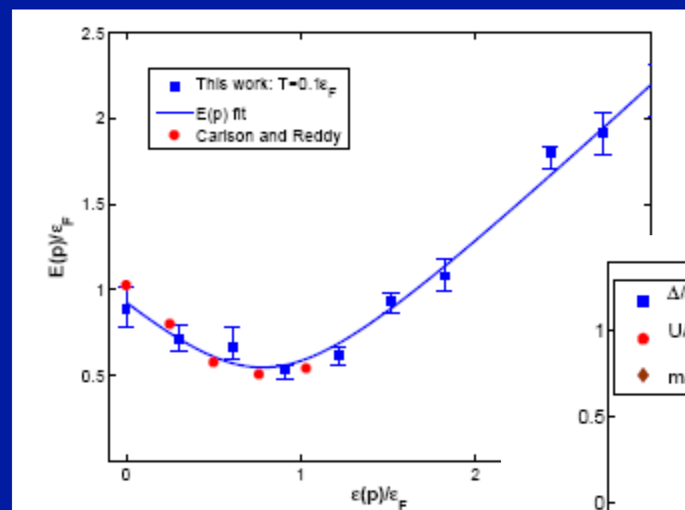


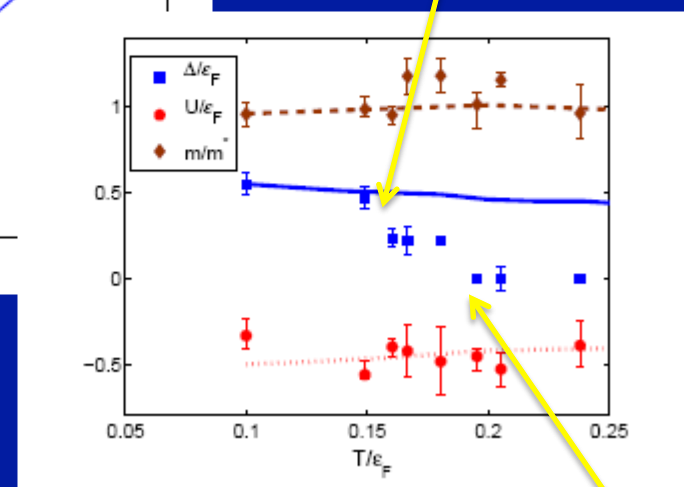
Data from Fischer *et al*, Rev. Mod. Phys. 79, 353 (2007)



$$G(p,\tau)=\frac{1}{Z}\mathrm{Tr}\left\{\exp\left[-(\beta-\tau)(H-\mu N)\right]\psi^{\dagger}(p)\times \exp\left[-\tau(H-\mu N)\right]\psi(p)\right\}$$

$$=-\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega A(p,\omega)\frac{\exp(-\omega\tau)}{1+\exp(-\omega\beta)}$$

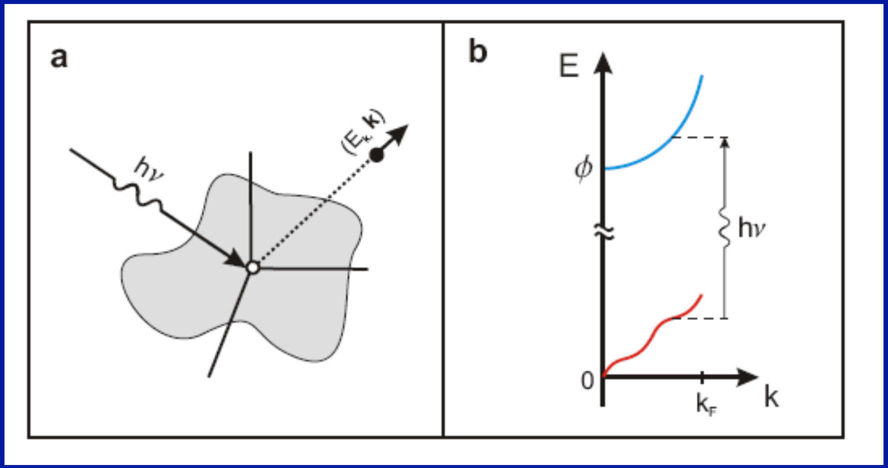


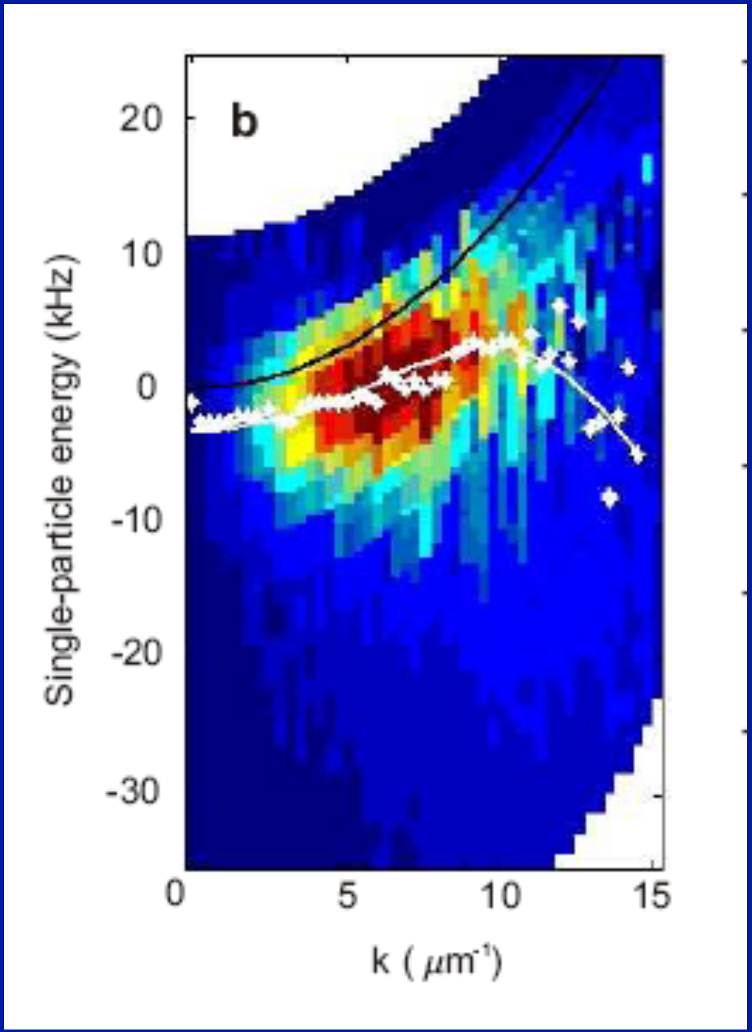


T_c

The pseudo-gap vanishes at T_0

Magierski, Wlazlowski, Bulgac, and Drut
Phys. Rev. Lett. <u>103</u>, 210403 (2009), arXiv:0801.1504





$$E(N) + h\nu = E(N-1) + E_k + \frac{\hbar^2 k^2}{2m} + \phi$$

Using photoemission spectroscopy to probe a strongly interacting Fermi gas
Stewart, Gaebler, and Jin, Nature, <u>454</u>, 744 (2008)

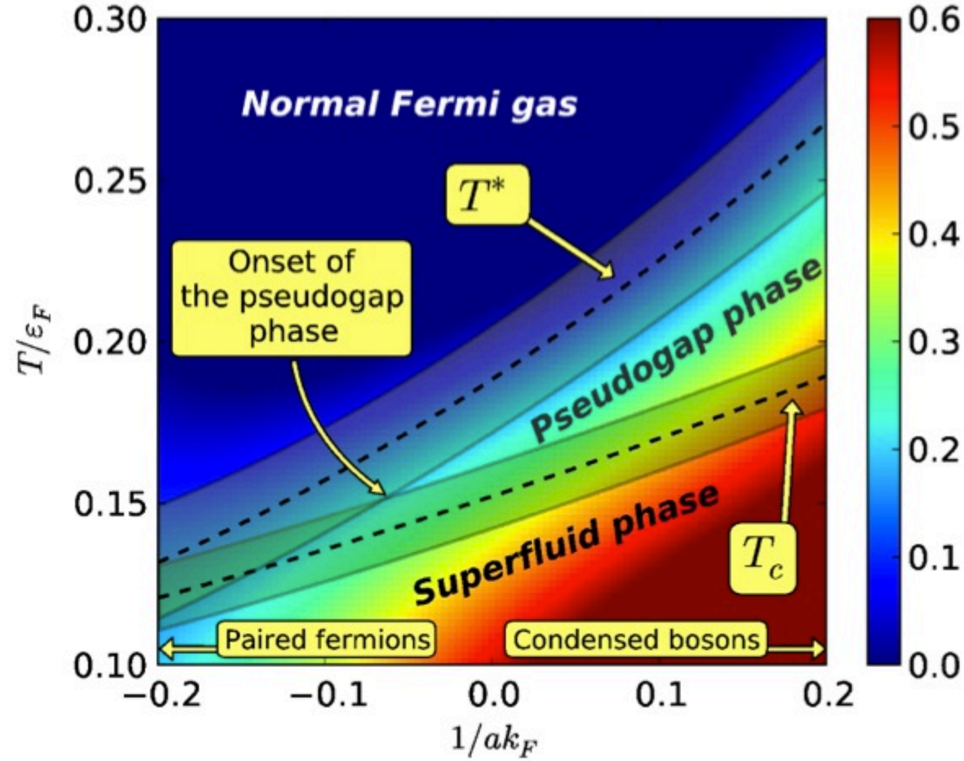


FIG. 3 (color online). The gap Δ/ε_F extracted from the spectral weight function as a function of temperature and scattering length. The dashed lines denote two temperatures: critical temperature T_c and the crossover temperature T^*. Uncertainties (both systematic and statistic errors, estimated to be no more than 10%) of these temperatures, are denoted by shaded area.

The onset of the pseudo-gap phase in ultracold Fermi gases

Magierski, Wlazlowski, and Bulgac, Phys. Rev. Lett. 107, 145304 (2011)

Shear viscosity of a unitary Fermi gas
(the only complete ab initio calculation in a Fermi system)

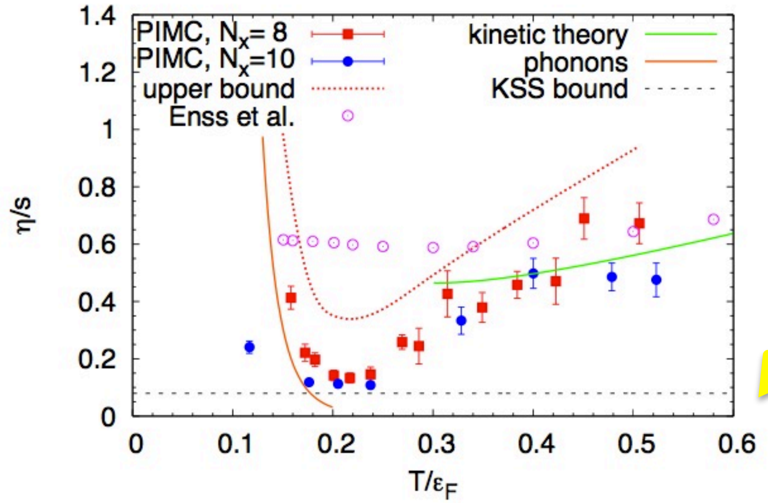


FIG. 3: (Color online) The ratio of the shear viscosity to the entropy density η/s as a function of dimensionless temperature for 8^3-lattice (red) squares and 10^3-lattice (blue) circles. The error bars only presents the stability of the combined (SVD and MEM) analytic continuation procedure with respect to the change of algorithm parameters, and do not include systematic errors of the entropy determination. By (red) dotted line conservative estimation for the upper bound is depicted. Result of the T-matrix theory are plotted by open (purple) circles [15]. In the high and low temperatures regime known asymptotics are depicted: for $T > 0.3\varepsilon_F$ by (green) line prediction of the kinetic theory and for $T < 0.2\varepsilon_F$ by (brown) line contribution from phonon excitations [13]. By dashed (black) line the KSS bound is plotted.

Lower limit for "perfect liquid"

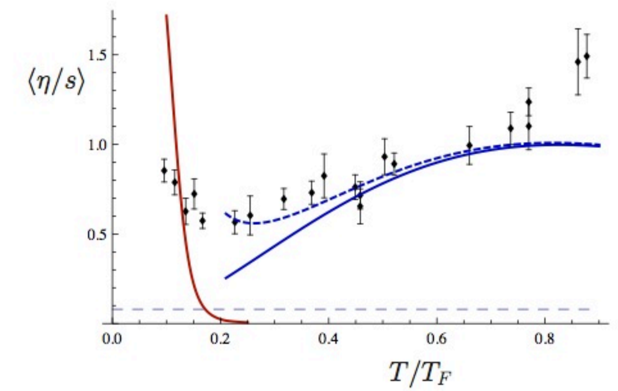


Fig. 11 Trap average $\langle \alpha_s \rangle = \langle \eta/s \rangle$ extracted from the damping of the radial breathing mode. The data points were obtained using equ. (91) to analyze the data published by Kinast et al. [9]. The thermodynamic quantities (S/N) and E_0/E_F were taken from [22]. The solid red and blue lines show the expected low and high temperature limits. Both theory curves include relaxation time effects. The blue dashed curve is a phenomenological two-component model explained in the text.

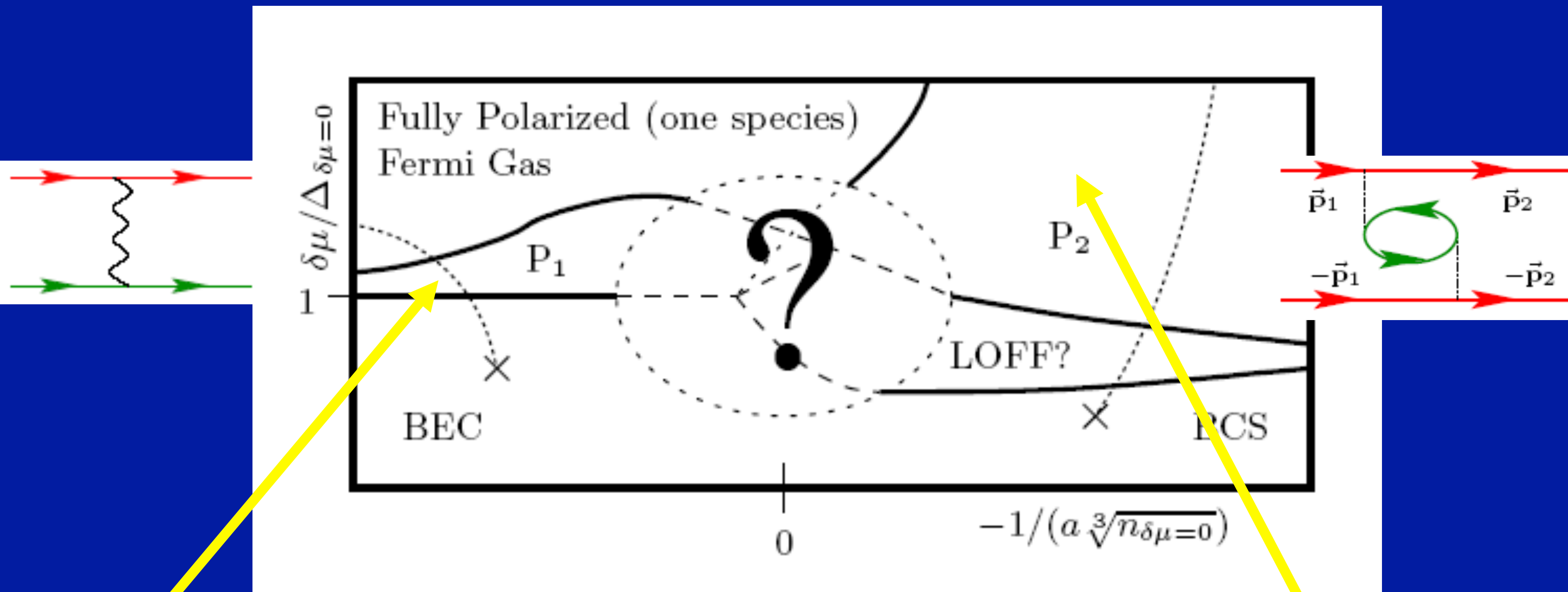
Schaefer and Chafin, chapter in *BCS-BEC crossover and Unitary Fermi Gas,* Lect. Notes in Phys. ed. Zwerger, Springer (2012)

Wlazlowski, Magierski, and Drut, Phys. Rev. Lett. 109, 020406 (2012)
Enss, Haussmann, and Zwerger, Ann. Phys. 326, 770 (2011)
Kovtun, Son, and Starinets (KSS), Phys. Rev. Lett. 94, 111601 (2005)

What is happening in spin imbalanced systems?

Induced P-wave superfluidity (*even though fermions interact in s-wave only)*

Two new superfluid phases where before they were not expected

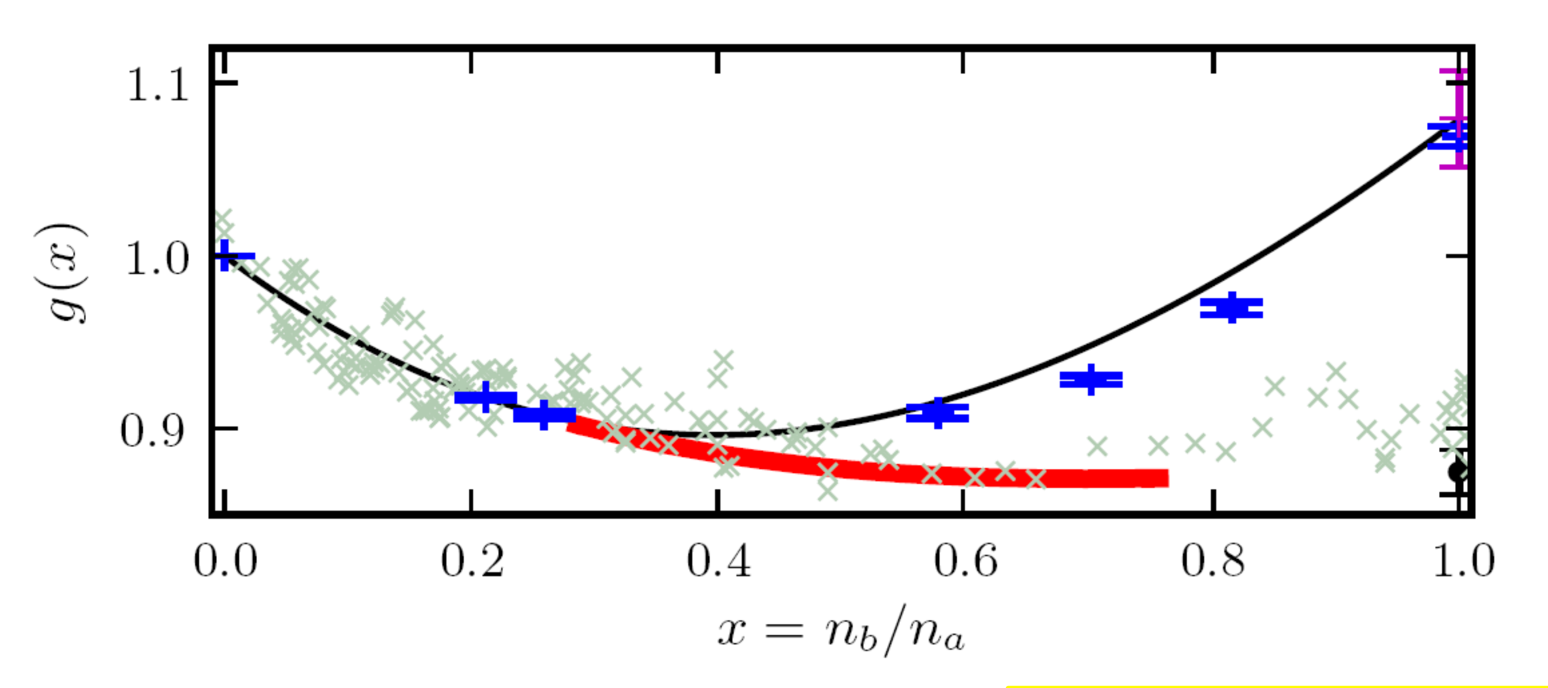


One Bose superfluid coexisting with one P-wave Fermi superfluid

Two coexisting P-wave Fermi superfluids

Bulgac, Forbes, and Schwenk, Phys. Rev. Lett. 97, 020402 (2006)

A refined Equation of State for spin imbalanced systems



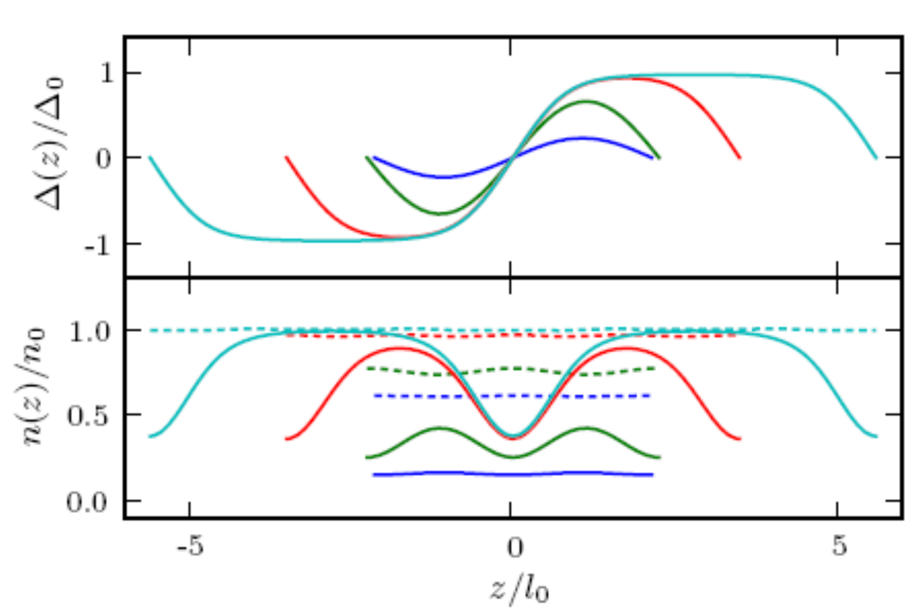
Bulgac and Forbes,
Phys. Rev. Lett. 101, 215301 (2008)

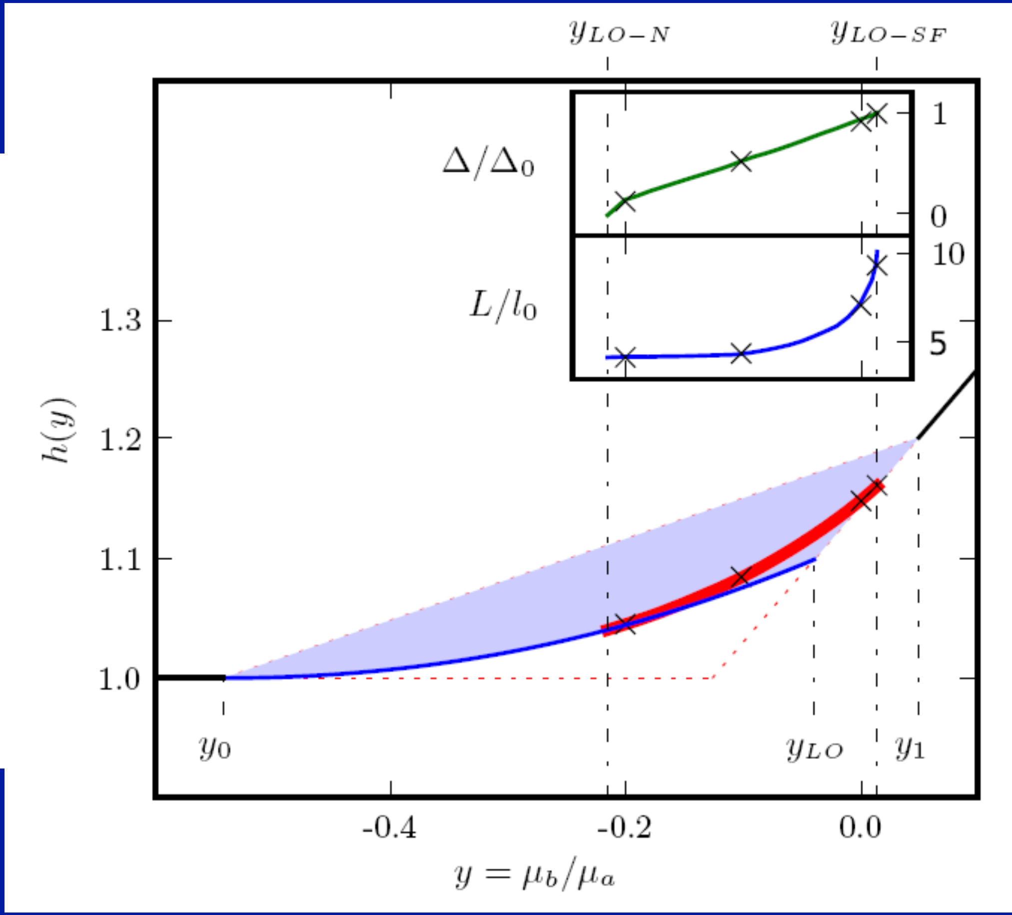
Red line: Larkin-Ovchinnikov phase

Black line: normal part of the energy density
Blue points: DMC calculations for normal state, Lobo et al, PRL 97, 200403 (2006)
Gray crosses: experimental EOS due to Shin, Phys. Rev. A 77, 041603(R) (2008)

$$E(n_a,n_b) = \frac{3}{5}\frac{(6\pi^2)^{2/3}\hbar^2}{2m}\left[n_a g\left(\frac{n_b}{n_a}\right)\right]^{5/3}$$

A Unitary Fermi Supersolid: the Larkin-Ovchinnikov phase



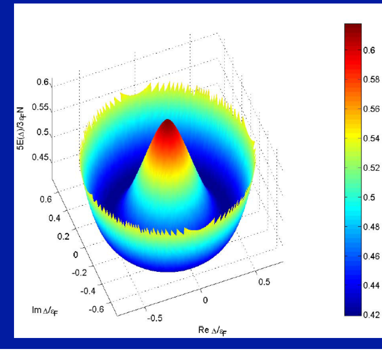


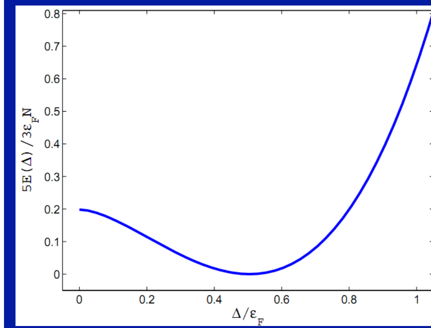
Bulgac and Forbes
Phys. Rev. Lett. 101, 215301 (2008)

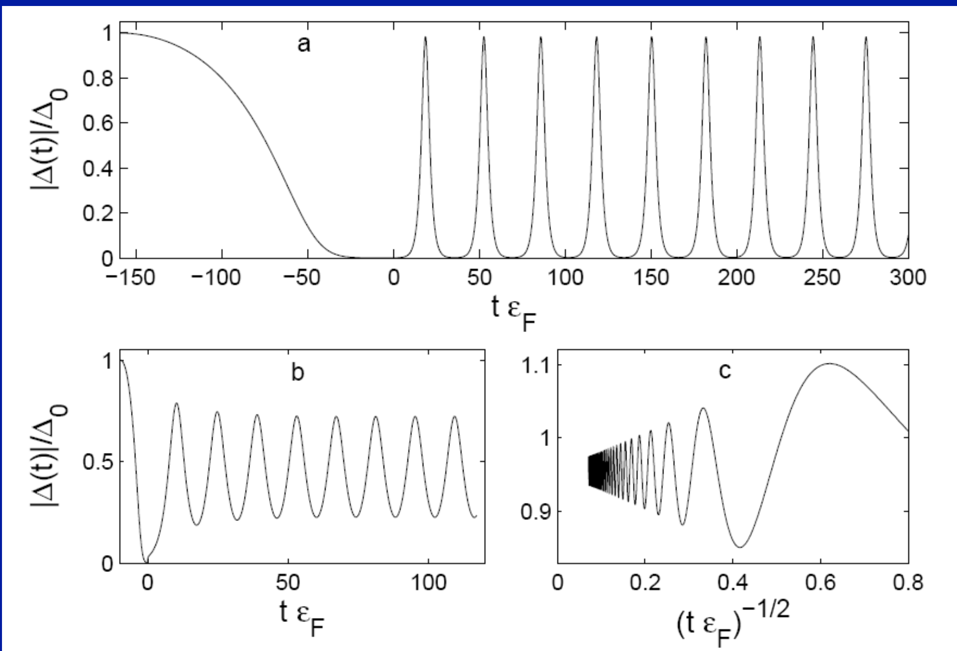
$$P[\mu_a,\mu_b] = \frac{2}{30\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left[\mu_a h\left(\frac{\mu_b}{\mu_a}\right)\right]^{5/2}$$

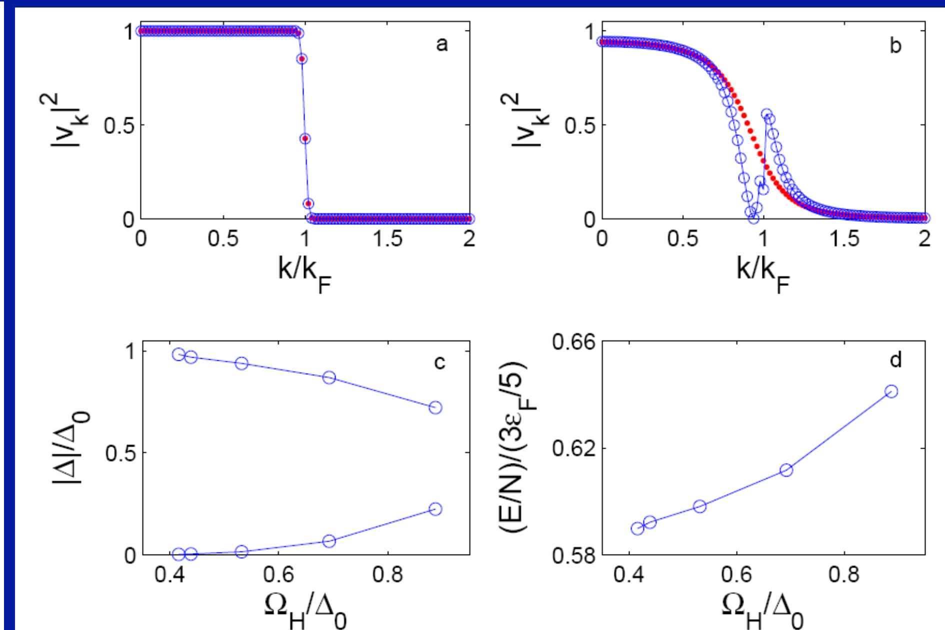
NB This is a gas system at the same time!

The Higgs mode









- All these modes have a very low frequency below the pairing gap, a very large amplitude and very large excitation energy

- None of these modes can be described either within two-fluid hydrodynamics or Landau-Ginzburg like approaches

Bulgac and Yoon, Phys. Rev. Lett. 102, 085302 (2009)

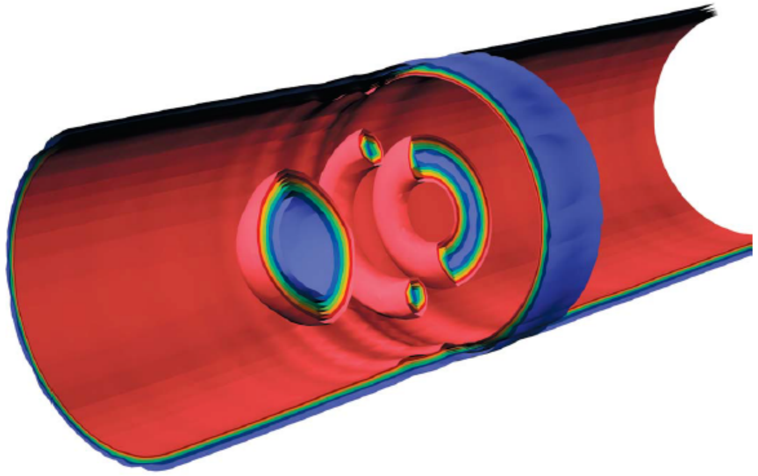


Fig. 2. A spherical projectile flying along the symmetry axis leaves in its wake two vortex rings.

Onset of quantum turbulence in a fermionic superfluid, conjectured by Feynman (1956

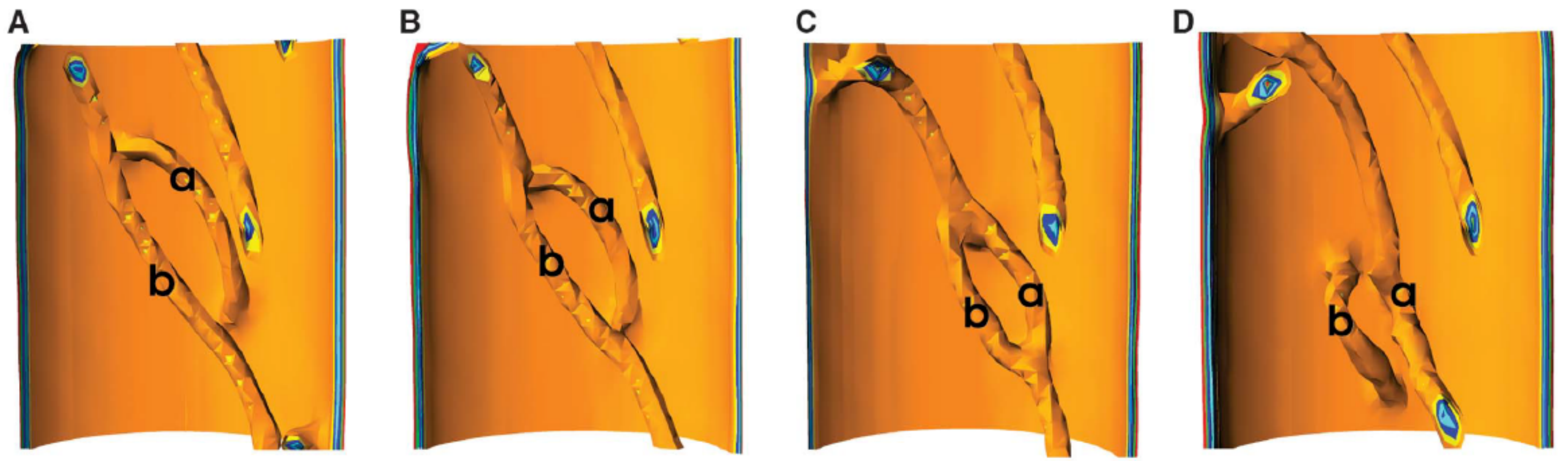
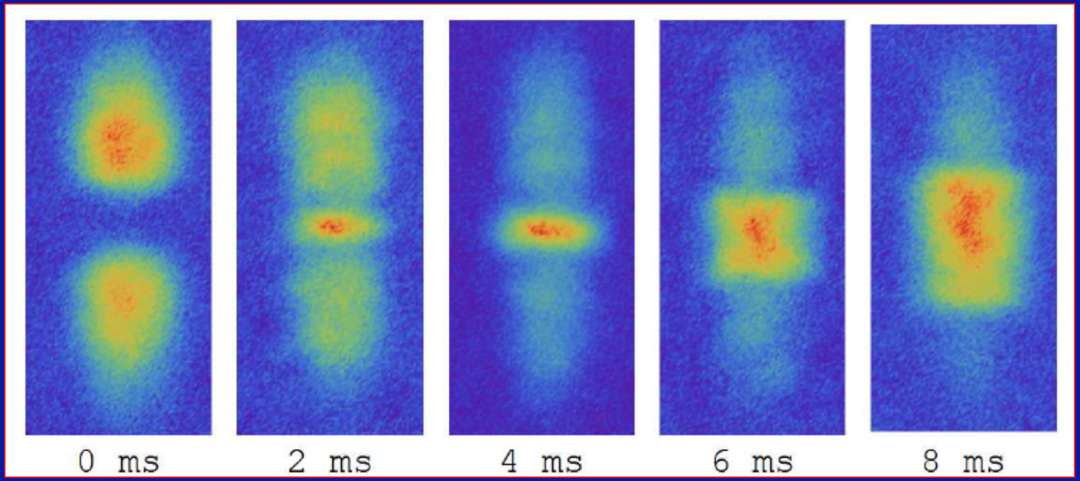


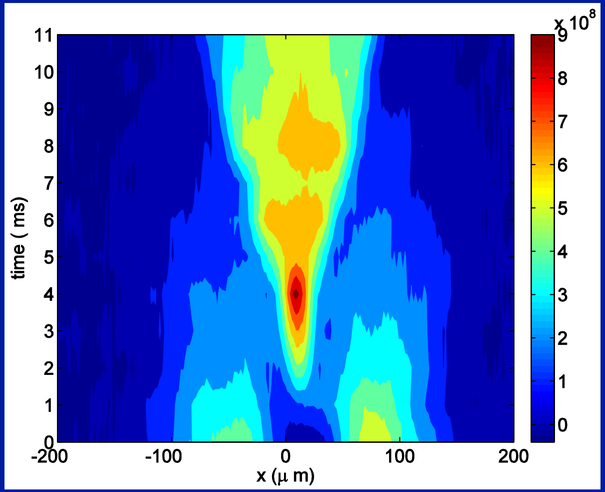
Fig. 3. (**A** to **D**) Two vortex lines approach each other, connect at two points, form a ring and exchange between them a portion of the vortex line, and subsequently separate. Segment (a), which initially belonged to the vortex line attached to the wall, is transferred to the long vortex line (b) after reconnection and vice versa.

Real-time dynamics of quantized vortices in a unitary Fermi gas

Bulgac, Luo, Magierski, Roche, and Yu, Science, 332, 1288(2011)

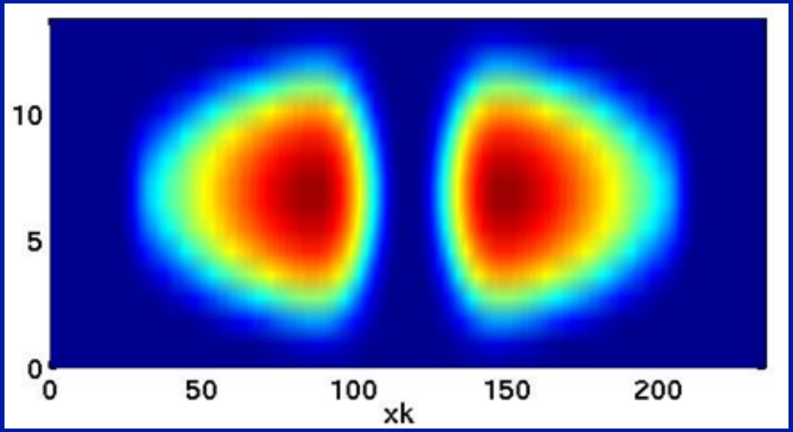
and about 4 hours of video at http://www.phys.washington.edu/groups/qmbnt/UFG

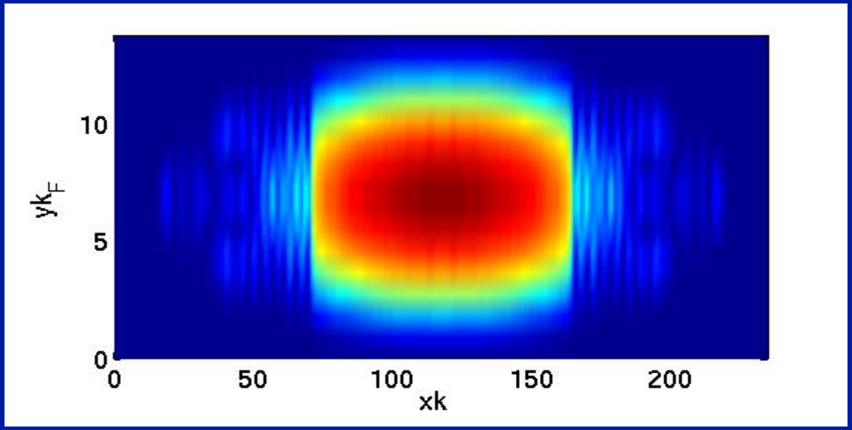




Observation of shock waves in a strongly interacting Fermi gas

Joseph, Thomas, Kulkarni, and Abanov, Phys. Rev. Lett. 106, 150401 (2011)

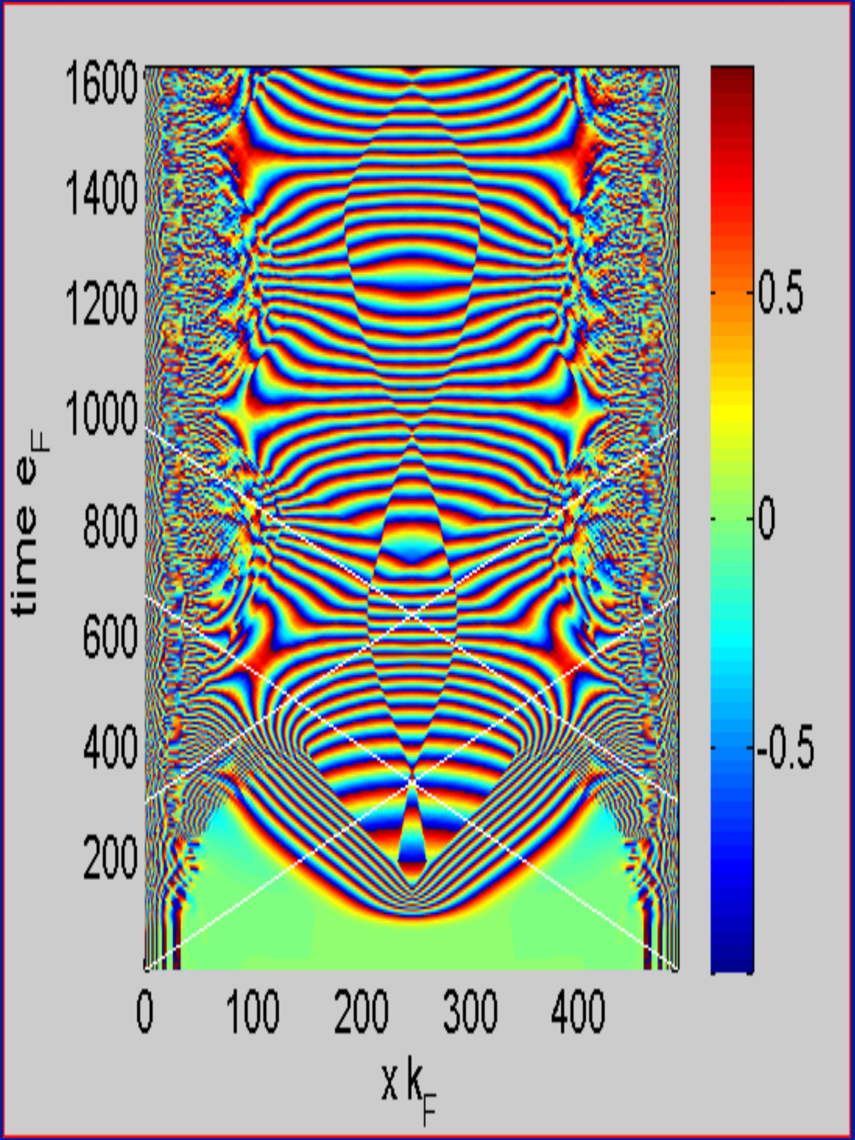




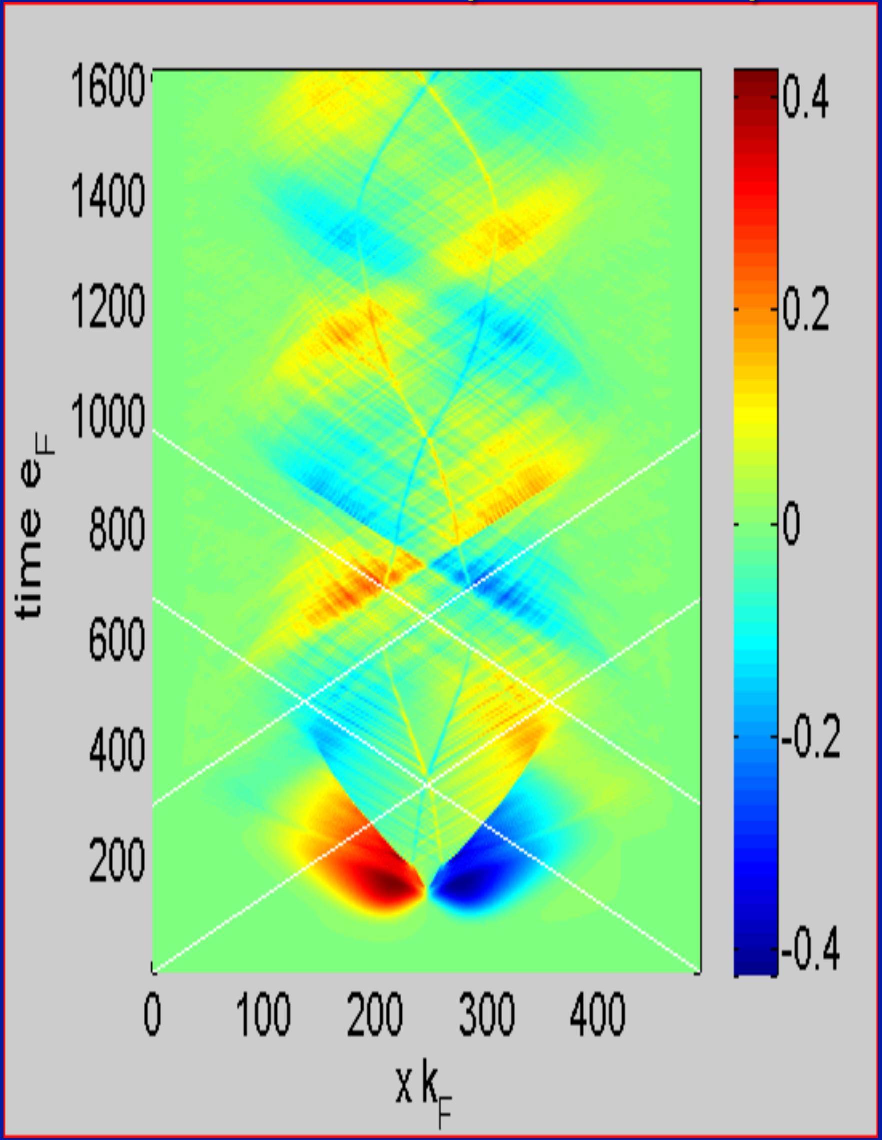
Number density of two colliding cold Fermi gases in TDSLDA

Bulgac, Luo, and Roche, Phys. Rev. Lett. 108, 150401 (2012)

Dark solitons/domain walls and shock waves in the collision of two UFG clouds (about 750 fermions, TDSLDA (superfluid extension of TDDFT) calculation)



Phase of the pairing gap normalized to ε_F



Local velocity normalized to Fermi velocity

Bulgac, Luo, and Roche, Phys. Rev. Lett. <u>108</u>, 150401 (2012)